Detection Inflection s-shaped model Using SPRT based on Order Statistics

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ABSTRACT- To assess the software reliability by statistical means yields efficient results. In this paper, for an effective monitoring of failure process we have opted Sequential Probability Ratio Test (SPRT) over the time between every r^{th} failure (r is a natural number >=2) instead of inter-failure times. This paper projects a controlling framework based on order statistics of the cumulative quantity between observations of time domain failure data using mean value function of Inflection S-Shaped Model. The two unknown parameters can be estimated using the Maximum Likelihood Estimation (MLE).

Keywords-SPRT, Software Quality, Time Domain Data, Order Statistics, Inflection S-Shaped model

I. INTRODUCTION

Order statistics deals with properties and applications of ordered random variables and functions of these variables. The use of order statistics is significant when inter failure time is less or failures are frequent. Let A denote a continuous random variable with probability density function (pdf), f(a) and cumulative distribution function (cdf), F(a), and let (A1 , A2 , …, Ak) denote a random sample of size k drawn on A. The original sample observations may be unordered with respect to magnitude. A transformation is required to produce a corresponding ordered sample. Let (A(1) , A(2) , …, A(k)) denote the ordered random sample such that A(1) < A(2) < … < A(k); then (A(1), A(2), …, A(k)) are collectively known as the order statistics derived from the parent A. The various distributional characteristics can be known from Bala Krishnan and Cohen(1991).

II.WALD'S SEQUENTIAL TEST FOR A POISSON PROCESS

The sequential probability ratio test (SPRT) was developed by A. Wald at Columbia University in 1943. Due to its usefulness in development work on military and naval equipment it was classified as "Restricted" by the Espionage Act [6]. A big advantage of sequential tests is that they require fewer observations (time) on the average than fixed sample size tests. SPRTs are widely used for statistical quality control in manufacturing processes. An SPRT for homogeneous Poisson processes is described below.

Let {N (t), t≥ 0} be a homogeneous Poisson process with rate 'λ'. In our case, N(t)=number of failures up to time 't' and 'λ' is the failure rate (failures per unit time). Suppose that we put a system on test (for example a software system, where testing is done according to a usage profile and no faults are corrected) and that we want to estimate its failure rate 'λ '. We cannot expect to estimate 'λ' precisely. But we want to reject the system with a high probability if our data suggest that the failure rate is larger than λ1 and accept it with a high probability, if it's smaller than λ_0 ($0 < \lambda_0 < \lambda_1$). As always with statically tests, there is some risk to get the wrong answers. So we have to specify two (small) numbers 'α' and 'β', where 'α' is the probability of falsely rejecting the system. That is rejecting the system even if $\lambda \leq \lambda_0$. This is the "producer's" risk. β is the probability of falsely accepting the system. That is accepting the system even if $\lambda \geq 1$. This is the "consumer's" risk. With specified choices of λ_0 and λ_1 such that $0 < \lambda_0 < \lambda_1$, the probability of finding N(t) failures in the time span (0,t) with λ_1, λ_0 as the failure rates are respectively given by

$$P_1 = \frac{e^{-\lambda_1}[\lambda_1 t]^{N(t)}}{N(t)!} \qquad (2.1)$$

$$P_0 = \frac{e^{-\lambda_0 t}[\lambda_0 t]^{N(t)}}{N(t)!} \qquad (2.2)$$

The ratio $\frac{P_1}{P0}$ at any time't' is considered at a measure of deciding the truth towards λ_0 or λ_1, given a sequence of time instants say $t_1 < t_2 < t_3 <\ldots\ldots\ldots< t_k$ and the ccorresponding realizations

N$(t_1), N(t_2)$, ……..N(t_k) of N(t). simplification of $\frac{p_1}{p_0}$ gives

$$\frac{p_1}{p_0} = \exp\,(\lambda_0 - \lambda_1)t + \left[\frac{\lambda_1}{\lambda_0}\right]^{N(t)} \quad (2.3)$$

The decision rule of SPRT is to decide in favor of λ_1 , in favor of λ_0 or to continue by observing the number of failures at a later time than 't' according as $\frac{p_1}{p_0}$ is greater than or equal to a constant say A, less than or equal to a constant say B or in between the constants A and B. That is, we decide the given software product as unreliable, reliable or continue the test process with one more observation in failure data, according as

$$\frac{p_1}{p_0} \geq A$$

$$\frac{p_1}{p_0} \leq B$$

$$B < \frac{p_1}{p_0} < A$$

The approximate values of the constants A and B are taken as

$$A \cong \frac{1-\beta}{\alpha}, \quad B \cong \frac{\beta}{1-\alpha}$$

Where α and $'\beta'$ are the risk probabilities as defined earlier. A simplified version of the above decision processes is to reject the system as unreliable if N(t) falls for the first time above the line

$$N_u(t) = a.t + b_2 \qquad (2.4)$$

to accept the system to be reliable if N(t) falls for the first time below the line

$$N_L(t) = a.t - b_1 \qquad (2.5)$$

To continue the test with one more observation on [t, N(t)] as the random graph of [t, N(t)] is between the two linear boundaries given by equations (2.4) and (2.5)

Where $\alpha = \frac{\lambda_1 - \lambda_0}{\log\left[\frac{\lambda_1}{\lambda_0}\right]}$

$$b_1 = \frac{\log\frac{1-\alpha}{\beta}}{\log\left[\frac{\lambda_1}{\lambda_0}\right]}$$

$$b_2 = \frac{\log\left[\frac{1-\beta}{\alpha}\right]}{\log\left[\frac{\lambda_1}{\lambda_0}\right]}$$

The parameters $\alpha, \beta, \lambda_0, \lambda_1$ can be chosen in several ways. One way suggested by [5] is

$$\lambda_0 = \frac{\lambda.\log(p)}{q-1}, \quad \lambda_1 = q.\frac{\lambda.\log q}{q-1} \quad \text{where } q = \frac{\lambda_1}{\lambda_0}$$

If λ_0 and λ_1 are chosen in this way, the slope of NU (t) and NL (t) equals λ. The other two ways of choosing λ_0 and $\lambda_1$are from past projects (for a comparison of the projects) and from part of the data to compare the reliability of different functional areas (components).

III. ILLUSTRATING THE MLE METHOD.

Based on the inter failure data given in Data set #1 & Data Set#2, we demonstrate the software failures process through failure control chart. We used cumulative time between failures data for software reliability monitoring. The use of cumulative quality is a different and new approach, which is of particular advantage in reliability. 'a' and 'b' are Maximum Likely hood Estimates (MLEs) of parameters 'a' and 'b' and the values can be computed using iterative method for the given cumulative time between failures data.

The probability density function of a two-parameter inflection S-shaped model has the form:

$$f(t) = \frac{be^{-bt}(1+\beta)}{(1+\beta e^{-bt})^2}$$

The corresponding cumulative distribution function is:

$$F(t) = = \frac{1}{1+\beta e^{-bt}}(1- e^{-bt})$$

Mean value function of the model is

$$m(t) = \frac{a}{1+\beta e^{-bt}}(1- e^{-bt})$$

For r^{th} order statistics, the mean value function is expressed as

$$m^r(t) = \left(\frac{a(1-e^{-bt})}{1+\beta e^{-bt}}\right)^r$$

The failure intensity function of r^{th} order is given as: $\lambda^r(t) = [\, m^r(t)\,]$

To estimate 'a' and 'b', for a sample of n units, first obtain the likelihood function:

The likelihood function

$$L = e^{-m^r(t_n)} \prod_{i=1}^{n} \lambda^r(t_i)$$

Take the natural logarithm on both sides, The Log Likelihood function is given as (Pham, 2006):

$$\text{Log } L = \sum_{i=1}^{n} \log[\lambda^r(t_i)] -$$

$$m^r(t_n) = \sum_{i=1}^{n} \log\left(\frac{a^r \; r\, b e^{-(bt_i)^{r-1}} (1-e^{-(bt_i)})^{r-1}(1+\beta)}{(1+\beta e^{-bt_i})^{r+1}}\right.$$

$$\left. - \frac{a^r[1-e^{-(bt_n)}]^r}{(1+\beta e^{-bt_n})^r}\right)$$

$$a^r = n\left(\frac{1+\beta e^{-bt_n}}{1-e^{-(bt_n)}}\right)^r \qquad (3.1)$$

The parameter 'a' is estimated by taking the partial derivative w.r.t 'a' and equating to '0'.

The parameter 'b' is estimated by iterative Newton Rap son Method using

$b_{n-1} = b_n - \frac{g(b_n)}{g'(b_n)}$, which is substituted in finding 'a'. Where g (b) and g'(b) are expressed as follows.

Taking the partial derivative w.r.t 'b' and equating to '0'.

$$g(b) = \frac{n}{b} + \sum_{i=0}^{n}\left(-t_i + \frac{t_i e^{-(bt_i)}}{1-e^{-(bt_i)}}(r-1) + \frac{t_i e^{-(bt_i)}}{1+\beta e^{-(bt_i)}} r(+1)\beta\right)$$

$$- \frac{nr\, t_n e^{-bt_n}(1+\beta)}{(1-e^{-bt_n})(1+\beta e^{-bt_n})} \qquad (3.2)$$

Again partially differentiating w.r.t 'b' and equating to 0

$$g'(b) = -\frac{n}{b^2}\sum_{i=0}^{n}(-t_i^{\,2}\, e^{-(bt_i)})\left[\frac{r-1}{(1-e^{-bt_i})^2}\right] + nrt_n^{\,2}(1+\beta)$$

$$\left[\frac{e^{-(bt_n)}\left((1-e^{-(bt_n)}) + e^{-(bt_n)}(1+\beta e^{-(bt_n)})\right)}{(1-e^{-bt_n})^2 (1-\beta e^{-bt_n})^2}\right] \qquad (3.3)$$

Iterative Newton-Raph son method is used to Solve the equations (3.1), (3.2), (3.3) in order to get the approximated values of a & b for the given failure data sets.

IV. SEQUENTIAL TEST FOR SOFTWARE RELIABILITY GROWTH MODELS

The Poisson process we know that the expected value of N(t) = λ t called the average number of failures experienced in time 't' .This is also called the mean value function of the Poisson process. On the other hand if we consider a Poisson process with a general function (not necessarily linear) m (t) as its mean value function the probability equation of a such a process is

$$P[N(t) = Y] = \frac{[m(t)]^y}{y!} \cdot e^{-m(t)}, y = 0,1,2,---$$

Depending on the forms of m (t) we get various Poisson processes called NHPP for our model the mean value function is Inflection S-Shaped:

$m(t) = a\left[\frac{1-e^{-bt}}{1+e^{-bt}}\right]$ where a>0, b>0, t>0

we may write

$p_1 = \frac{e^{-m_1(t)}.[m_{1(t)}]^{N(t)}}{N(t)!}$

$p_0 = \frac{e^{-m_0(t)}.[m_{0(t)}]^{N(t)}}{N(t)!}$

Where $m_1(t), m_0(t)$, are values of the mean value function at specified sets of its parameters indicating reliable software and unreliable software respectively. For instance the model we have been considering their m(t) function, contains a pair of parameters a, b with ‘a’ as a multiplier. Also a, b are positive. Let $p_{0,}\ p_1$, be values of the NHPP at two specifications of b say b_0, b_1 ($b_0 < b_1$) respectively. It can be shown that for our model m(t) at b1 is greater than that at b0. Symbolically $m_0(t) < m_1(t)$. Then the SPRT procedure is as follows:

Accept the system to be reliable $\frac{p_1}{p_0} \le B$

i.e. $\frac{e^{-m_1(t)}.[m_1(t)]^{N(t)}}{e^{-m_0(t)}.[m_0(t)]^{N(t)}} \le B$

i.e. $N(t) \le \frac{\log\left(\frac{\beta}{1-\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)}$ (4.1)

Decide the system to be unreliable and reject if $\frac{p_1}{p_0} \ge A$

i.e. $N(t) \ge \frac{\log\left(\frac{1-\beta}{\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)}$ (4.2)

Continue the test procedure as long as

$$\frac{\log\left(\frac{\beta}{1-\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} < N(t) < \frac{\log\left(\frac{1-\beta}{\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \quad (4.3)$$

Substituting the appropriate expressions of the respective mean value functions – m (t) of Inflection S-Shaped we get the decision rules and is given in followings lines

m (t)$=\alpha\left[\frac{1-e^{-bt}}{1+e^{-bt}}\right]$ where a>0, b>0 and t>0

Acceptance regions:

$$N(t) \le \frac{\log\left(\frac{\beta}{1-\alpha}\right) + a\left[\frac{\left[e^{-b_0 t} - e^{-b_1 t} + \beta\left(e^{-b_0 t} - e^{-b_1 t}\right)\right]}{\left(1+\beta e^{-b_1 t}\right)\left(1+\beta e^{-b_0 t}\right)}\right]}{\log\left[\left(\frac{1-e^{-b_1 t}}{1+\beta e^{-b_1 t}}\right)\left(\frac{1+\beta e^{-b_0 t}}{1-e^{-b_0 t}}\right)\right]} \quad (4.4)$$

Rejection region:

$$N(t) \ge \frac{\log\left(\frac{1-\beta}{\alpha}\right) + a\left[\frac{e^{-b_0 t} - e^{-b_1 t} + \beta\left(e^{-b_0 t} - e^{-b_1 t}\right)}{\left(1+\beta e^{-b_1 t}\right)\left(1+\beta e^{-b_0 t}\right)}\right]}{\log\left[\left(\frac{1-e^{-b_1 t}}{1+\beta e^{-b_1 t}}\right)\left(\frac{1+\beta e^{-b_0 t}}{1-e^{-b_0 t}}\right)\right]} \quad (4.5)$$

Continuation region:

$$\frac{\log\left(\frac{\beta}{1-\alpha}\right) + a\left[\frac{\left[e^{-b_0 t} - e^{-b_1 t} + \beta\left(e^{-b_0 t} - e^{-b_1 t}\right)\right]}{\left(1+\beta e^{-b_1 t}\right)\left(1+\beta e^{-b_0 t}\right)}\right]}{\log\left[\left(\frac{1-e^{-b_1 t}}{1+\beta e^{-b_1 t}}\right)\left(\frac{1+\beta e^{-b_0 t}}{1-e^{-b_0 t}}\right)\right]} <$$

$$N(t) < \frac{\log\left(\frac{1-\beta}{\alpha}\right) + a\left[\frac{e^{-b_0 t} - e^{-b_1 t} + \beta\left(e^{-b_0 t} - e^{-b_1 t}\right)}{\left(1+\beta e^{-b_1 t}\right)\left(1+\beta e^{-b_0 t}\right)}\right]}{\log\left[\left(\frac{1-e^{-b_1 t}}{1+\beta e^{-b_1 t}}\right)\left(\frac{1+\beta e^{-b_0 t}}{1-e^{-b_0 t}}\right)\right]}$$

(4.6)

It may be noted that in the above two models the decision rules are exclusively based on the strength of the sequential procedure (α, β) and the values of the respective mean value functions namely $m_0(t)$. $m_1(t)$. If the mean value function is linear in ‘t’ passing through origin, that is, m (t) = λ t the decision rules become decision lines as described by [5]. In that sense equations (4.1), (4.2), (4.3) can be regarded as generalizations to the decision procedure of [5].The applications of these results for live software failure data are presented with analysis in Section 5.

V. SPRT ANALYSIS OF LIVE DATA SETS

We see that the developed SPRT methodology is for a software failure data which is of the form [t, N(t)] where N(t) is the observed number of failures of software system or its sub system in ‘t’ units of time. In this section we evaluate the decision rules based on the considered mean value functions for two different data sets of the above form, borrowed from [2] [7] [8]. Based on the estimates of the parameter ‘b’ in each mean value function, we have chosen the specifications of b0=b-δ, b1=b+ δ equidistant on either side of estimate of b obtained through a Data Set to apply SPRT such that b0 < b < b1. Assuming the value of δ =0.000002, the choices are given in the following table.

5.1 Time domain data sets for ordered statistics

Data Set #1, #2: The Real-time Control System Data

The data sets were listed in "DATA" directory Containing 45 industry project failure data sets in the Handbook of Software Reliability Engineering (Lyu, 1996).

Table 5.1: Data Set #1

F No	TBF	F NO	TBF	F NO	TBF	F NO	TB F
1	760	33	87	65	276	97	15
2	758	34	19	66	1	98	1960
3	33	35	29	67	999	99	60
4	6	36	0	68	30	100	19
5	22	37	5	69	495	101	20
6	14	38	360	70	472	102	79
7	42	39	10	71	344	103	24
8	4	40	11	72	550	104	1737
9	84	41	100	73	131	105	7984
10	15	42	252	74	47	106	10
11	221	43	460	75	92	107	20
12	14	44	179	76	863	108	338
13	15	45	3	77	991	109	250
14	41	46	24	78	35	110	1682
15	1	47	253	79	9549	111	212
16	153	48	163	80	249	112	287
17	409	49	54	81	607	113	56
18	54	50	137	82	83	114	4973
19	24	51	328	83	614	115	3500
20	44	52	3	84	352	116	59
21	180	53	9	85	673	117	98
22	397	54	12	86	4179	118	2439
23	19	55	18	87	111	119	1812
24	145	56	9	88	75	120	6203
25	36	57	75	89	407	121	385
26	54	58	15	90	288	122	3500
27	1337	59	366	91	894	123	4892
28	163	60	428	92	1314	124	687
29	8	61	212	93	845	125	62
30	1	62	115	94	55	126	2796
31	17	63	264	95	409	127	3268
32	16	64	269	96	36	128	3845

Table 5.2: Data Set #2

FN O	TBF	FN O	TBF	FN O	TBF	FNO	TBF
1	3	35	227	69	529	103	108
2	30	36	65	70	379	104	0
3	113	37	176	71	44	105	3110
4	81	38	58	72	129	106	1247
5	115	39	457	73	810	107	943
6	9	40	300	74	290	108	700
7	2	41	97	75	300	109	875
8	91	42	263	76	529	110	245
9	112	43	452	77	281	111	729
10	15	44	255	78	160	112	1897
11	138	45	197	79	828	113	447
12	50	46	193	80	1011	114	386
13	77	47	6	81	445	115	446
14	24	48	79	82	296	116	122
15	108	49	816	83	1755	117	990
16	88	50	1351	84	1064	118	948
17	670	51	148	85	1783	119	1082
18	120	52	21	86	860	120	22
19	26	53	233	87	983	121	75
20	114	54	134	88	707	122	482
21	325	55	357	89	33	123	5509
22	55	56	193	90	868	124	100
23	242	57	236	91	724	125	10
24	68	58	31	92	2323	126	1071
25	422	59	369	93	2930	127	371
26	180	60	748	94	1461	128	790
27	10	61	0	95	843	129	6150
28	1146	62	232	96	12	130	3321
29	600	63	330	97	261	131	1045
30	15	64	365	98	1800	132	648
31	36	65	1222	99	865	133	5485
32	4	66	543	100	1435	134	1160
33	0	67	10	101	30	135	1864
34	8	68	16	102	143	136	4116

5.2. Data Analysis Using SPRT

In this section we evaluate the decision rules based on the given mean value function for two different data sets. Based on the estimated value of the parameter 'b', we have chosen the specifications of b0, b1 that are to be equidistant such that b0< b < b1. The choices are given in the following table.

Table 5.3: Specifications of b0, b1 for Data set # I

Order	Estimat e of a	Estimate of b	b0	b1
4^{th}	2.49733	0.000005	0.000003	0.000007
5^{th}	1.99883	0.000007	0.000005	0.000009

Table 5.4: Specifications of b0, b1 for Data Set # II

Order	Estima te of a	Estimate of b	b0	b1
4^{th}	4.9134 8	0.000008	0.000006	0.00001
5^{th}	3.7775 9	0.000009	0.000007	0.000011

Using the selected b0, b1 and $m_0(t)$, $m_1(t)$ we have calculated the decision rules given by Equations (4.4), (4.5), sequentially at each 't' of the data sets taking the strength (α, β) as (0.05,0.05). These are presented for the model in Table 5.8.

Table 5.4: 4^{th} order statistics for table 5.1 data set # 1

FNO	CFT	FNO	CFT	FNO	CFT
1	1557	13	7564	23	34077
2	1639	14	7612	24	35422
3	1973	15	8496	25	37476
4	2183	16	9356	26	39336
5	2714	17	10662	27	47688
6	3455	18	12523	28	50119
7	5045	19	13656	29	58707
8	5087	20	24480	30	69259
9	5222	19	13656	31	78723
10	5608	20	24480	32	88694
11	6599	21	26136		
12	7042	22	31174		

Table 5.5: 5th order statistics for table 5.1 data set # 1

FNO	CFT	FNO	CFT	FNO	CFT
1	1579	11	7603	21	47320
2	1738	12	8496	22	49620
3	2030	13	9632	23	58448
4	2714	14	11629	24	69259
5	3491	15	12793	25	78785
6	5054	16	24480		
7	5222	17	26809		
8	5608	18	31869		
9	6602	19	35386		
10	7233	20	37476		

Table 5.6: 4th order statistics for table 5.2 data set # 2

FNO	CFT	FNO	CFT	FNO	CFT
1	227	13	10258	25	42015
2	444	14	11175	26	42296
3	759	15	12559	27	48296
4	1056	16	13486	28	52042
5	1986	17	15277	29	53443
6	2676	18	16358	30	56485
7	4434	19	18287	31	62651
8	5089	20	20567	32	64893
9	5389	21	24127	33	76057
10	6380	22	28460	34	88682
11	7447	23	32408		
12	7922	24	37654		

Table 5.7: 5th order statistics for table 5.2 data set # 2

FNO	CFT	FNO	CFT	FNO	CFT
1	342	10	10089	19	37642
2	571	11	10982	20	42015
3	968	12	12559	21	45406
4	1986	13	14708	22	49416
5	3098	14	16185	23	53321
6	5049	15	17758	24	56485
7	5324	16	20567	25	62661
8	6380	17	25910	26	74364
9	7644	18	29361	27	84566

Table 5.8: SPRT Analysis for Inflection S-Shaped model

Data set	Order	T	N(t)	Acceptance Region(≤)	Rejection Region (≥)	Decision
1	4	1557	1	-3.469260	3.504097	Rejected
		1639	2	-3.468960	3.505623	
		1973	3	-3.467736	3.511835	
		2183	4	-3.466971	3.515739	
	5	1579	1	-5.013463	5.054164	Rejected
		1738	2	-5.013897	5.058674	
		2030	3	-5.014706	5.066956	
		2714	4	-5.016655	5.086355	
		3491	5	-5.018959	5.108388	
		5054	6	-5.023889	5.152702	
2	4	227	1	-5.760406	5.777027	Rejected
		444	2	-5.756913	5.789398	
		759	3	-5.751876	5.807344	
		1056	4	-5.747162	5.824250	
		1986	5	-5.732621	5.877102	
		2676	6	-5.722048	5.916232	
	5	342	1	-6.512516	6.534263	Rejected
		571	2	-6.511242	6.547517	
		968	3	-6.509088	6.570483	
		1956	4	-6.503876	6.629307	
		3098	5	-6.498693	6.693458	
		5049	6	-6.490883	6.805760	
		5324	7	-6.489914	6.821565	

VI. Conclusion

In this paper we have monitored two failure live data sets using SPRT. We are greatly succeeded in applying SPRT analysis over order statistic approach. We have observed that through order statistic approach we can have an early decision about acceptance/rejection of the software system being tested.

REFERENCES

[1]. Balakrishnan. N., Clifford Cohen. A., (1991). “Order Statistics and Inference”, Academic Press, INC. page 13.
[2]. Macgregor, J.F., Kourti, T., (1995). “Statistical process control of multivariate processes”. Control Engineering Practice Volume 3, Issue 3, 403-414.
[3]. Musa, J.D., Iannino, A., Okumoto, k., (1987). “Software Reliability: Measurement Prediction Application”. McGraw-Hill, New York.
[4]. Ohba, M., (1984). “Software reliability analysis model”. IBMJ. Res. Develop. 28. 428-443.
[5]. Pham. H., (2006). “System software reliability”, Springer.
[6]. Swapna S. Gokhale and Kishore S.Trivedi, (1998). “Log-Logistic Software Reliability Growth Model”. The 3rd IEEE International Symposium on High-Assurance Systems Engineering. IEEE Computer Society.
[7]. Lyu, M. R., (1996). “Handbook of Software Reliability Engineering”. McGraw-Hill publishing, ISBN 0-07-039400-8.
[8]. Oakland. J., (2008). “Statistical Process Control”, Sixth Edition, Butterworth-Heinemann, Elsevier.
[9]. Malaiya, Y. K., Sumitsur, Karunanithi, N. and Sun, Y. (1990). “Implementation considerations for software reliability”, 8th Annual Software Reliability Symposium, June 1.